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Precipitating Quasi-Geostrophic Equations

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Precipitating Quasi-Geostrophic Equations

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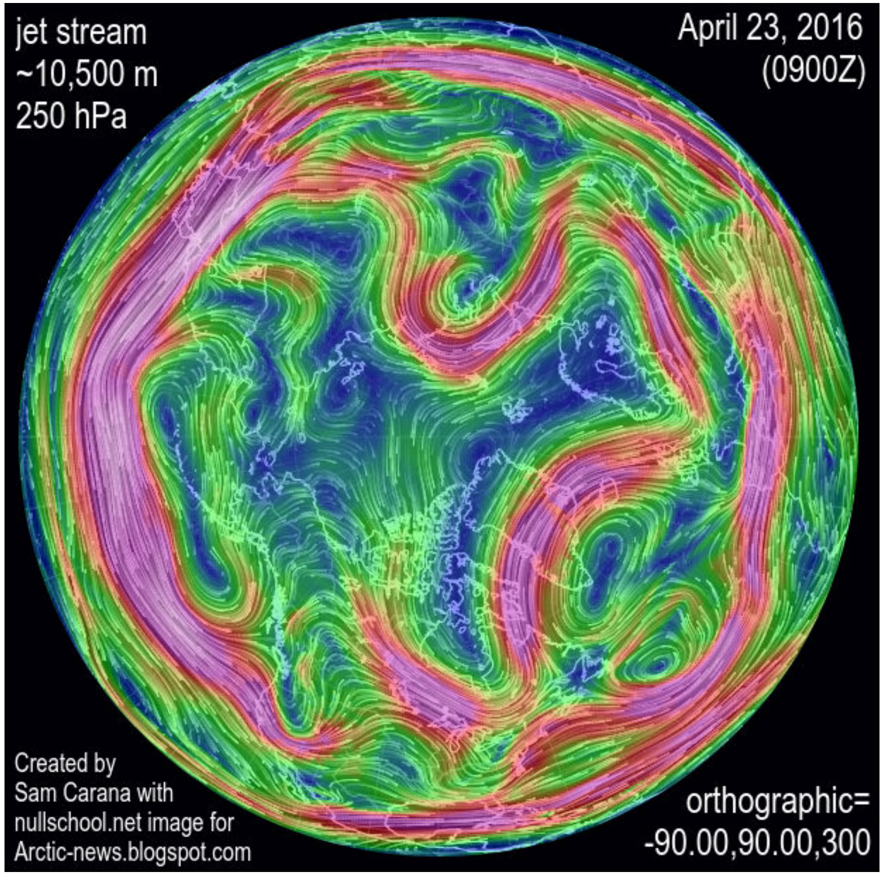

polar and subtropical jets merge over the Pacific

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Goals

Derive/analyze a precipitating QG model for large-scale mid-latitudes ==> effects of latent heat release for storms

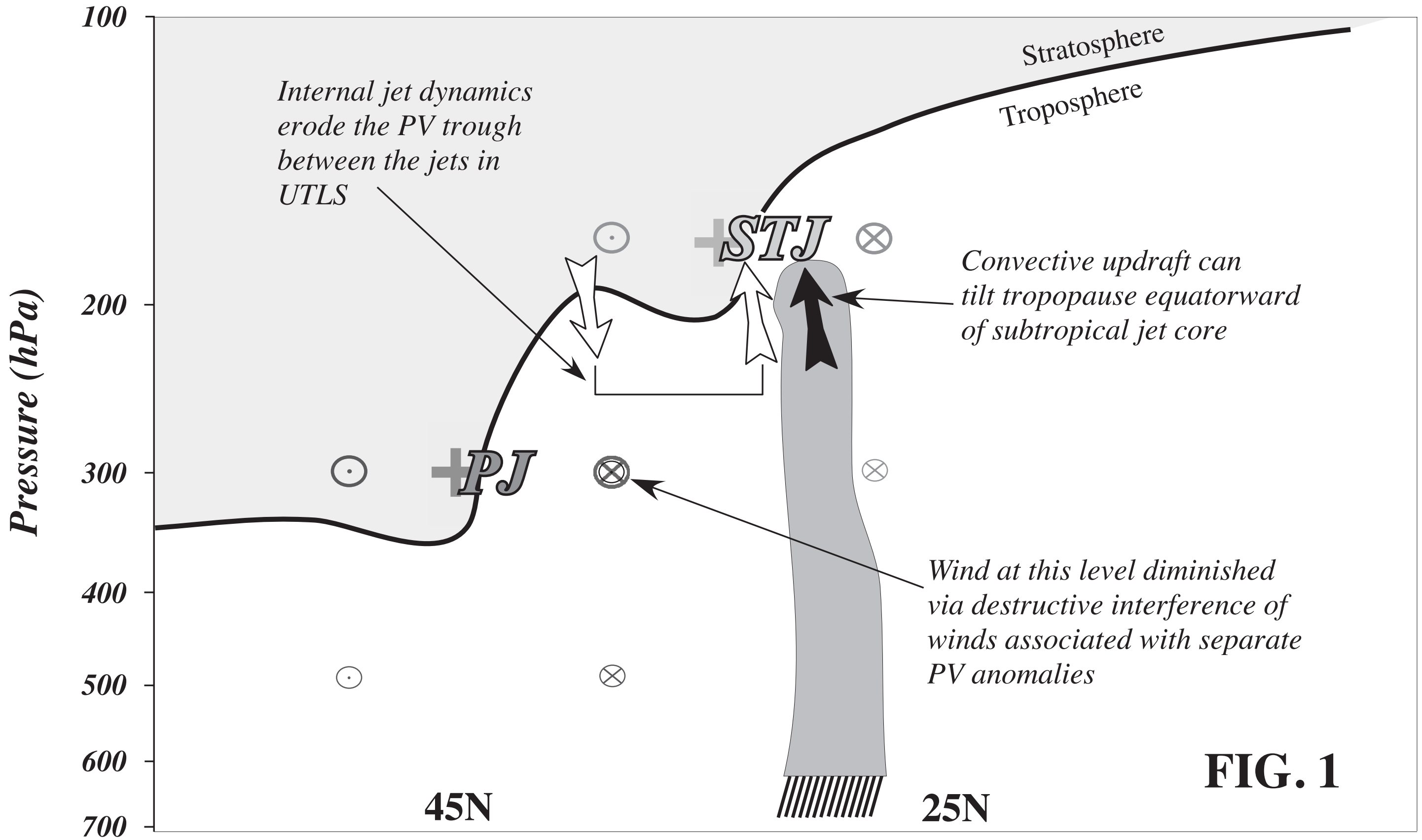


FIG. 1

Figure by Jon Martin/Andrew Winters

Outline

- The minimal FARE mode

  Boussinesq Core with Linearized Thermodynamics

  Asymptotically Fast Cloud Microphysics
  (condensation, evaporation, auto-conversion)

- Derivation of PQG from FARE
- Eady Baroclinic Instability Analysis
- Ongoing Work and Open Problems

$$\frac{D\mathbf{u}}{Dt} + f\sin(\phi)\mathbf{u}_h^{\perp} = -\nabla p + \mathbf{k}\, g\left(\frac{\theta}{\theta_o} + R_{vd}q_v - q_c - q_r\right)$$

$$\nabla\cdot\mathbf{u} = 0, \qquad \frac{D\theta}{Dt} + w\frac{d\tilde{\theta}(z)}{dz} = \frac{L}{c_p}(C_d - E_r)$$

$\theta^{\mathrm{tot}}(\mathbf{x},t) = \tilde{\theta}(z) + \theta(\mathbf{x},t)$, etc.

Bulk Cloud Physics

$$\frac{Dq_v}{Dt} = -C_d + E_r, \qquad \frac{Dq_c}{Dt} = C_d - A_r - C_r$$

$$\frac{Dq_r}{Dt} - \frac{\partial}{\partial z}(V_T q_r) = A_r + C_r - E_r$$

C_d: Condensation $q_v \to q_c$, E_r: Evaporation $q_r \to q_v$

A_r, C_r: Auto-conversion and Collection $q_c \to q_r$

$0 \leq V_T \lesssim 10 \text{ m s}^{-1}$: Rainfall velocity

All source terms need to be closed

For example:

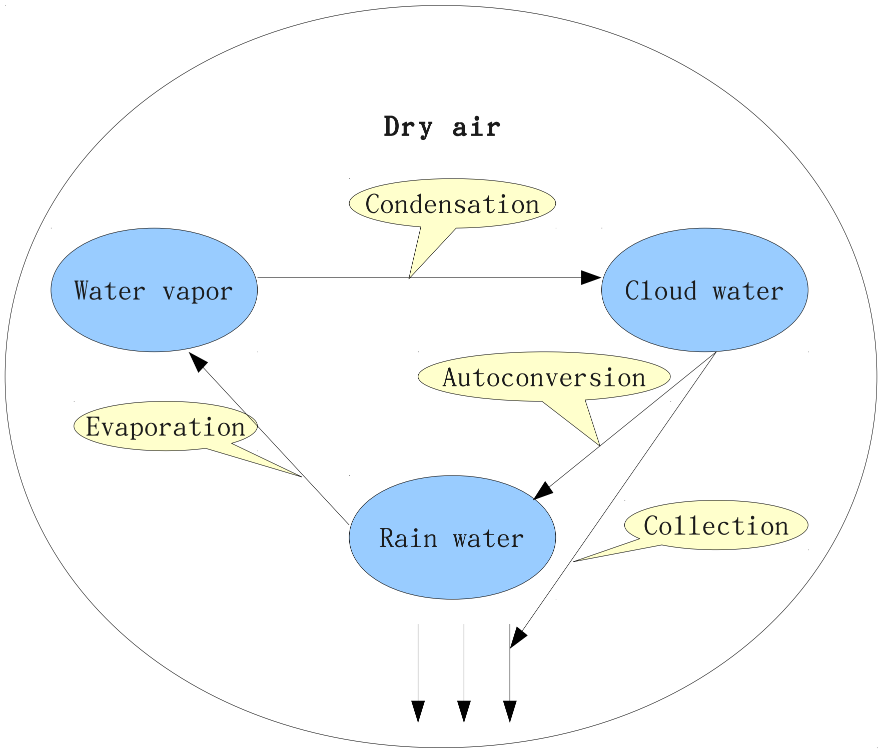
$$C_d = \alpha_d^{-1}(q_v - q_{vs}(T,p))^+$$

$$E_r = \alpha_r^{-1}(q_{vs}(T,p) - q_v)^+ q_r$$

$q_{vs}(T,p) \approx q_{vs}(z)$ at lowest order in a Taylor Series

$\alpha_d = \alpha_r$ are cond/evap time scale

Fast Cloud Microphysics



fast autoconversion: (Seitter & Kuo ’83, Emanuel ’86, Majda, Xing & Mohammadian ’10, Deng, S & Madja ’12)

Fast Autoconversion and Rain Evaporation: FARE

$$\frac{Dq_{\mathrm{tot}}}{Dt} - \frac{\partial}{\partial z}(V_T q_r) = 0$$

and the source terms maintain the constraint

$$C_d - E_r = \begin{cases} 0, & q_{\mathrm{tot}}\ (= q_v) < q_{vs} \\ -w\ dq_{vs}(z)/dz, & q_{\mathrm{tot}}\ (= q_{vs} + q_r) \geq q_{vs} \end{cases}$$

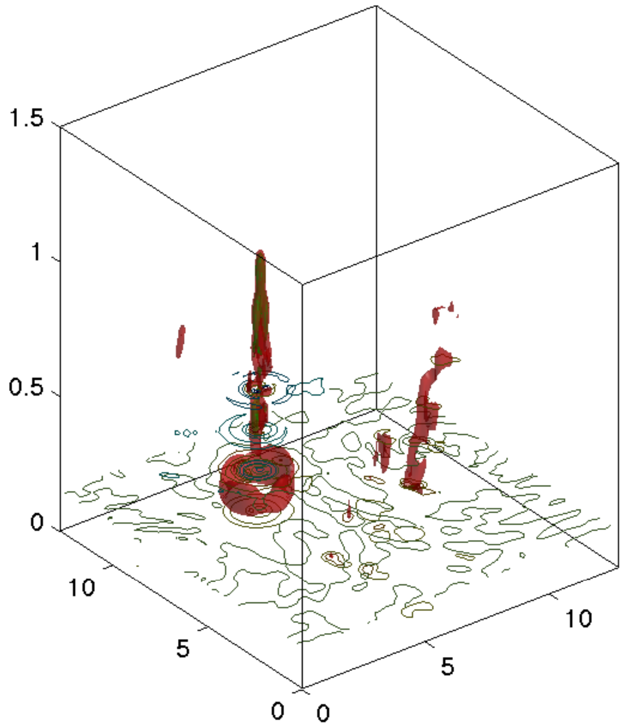
$q_{vs}(T,p) \approx q_{vs}(z)$ is the saturation profile

With $C_d - E_r > 0$:

$$\frac{D\theta}{Dt} + \frac{d\tilde{\theta}}{dz}w = \frac{L}{c_p}(C_d - E_r)$$

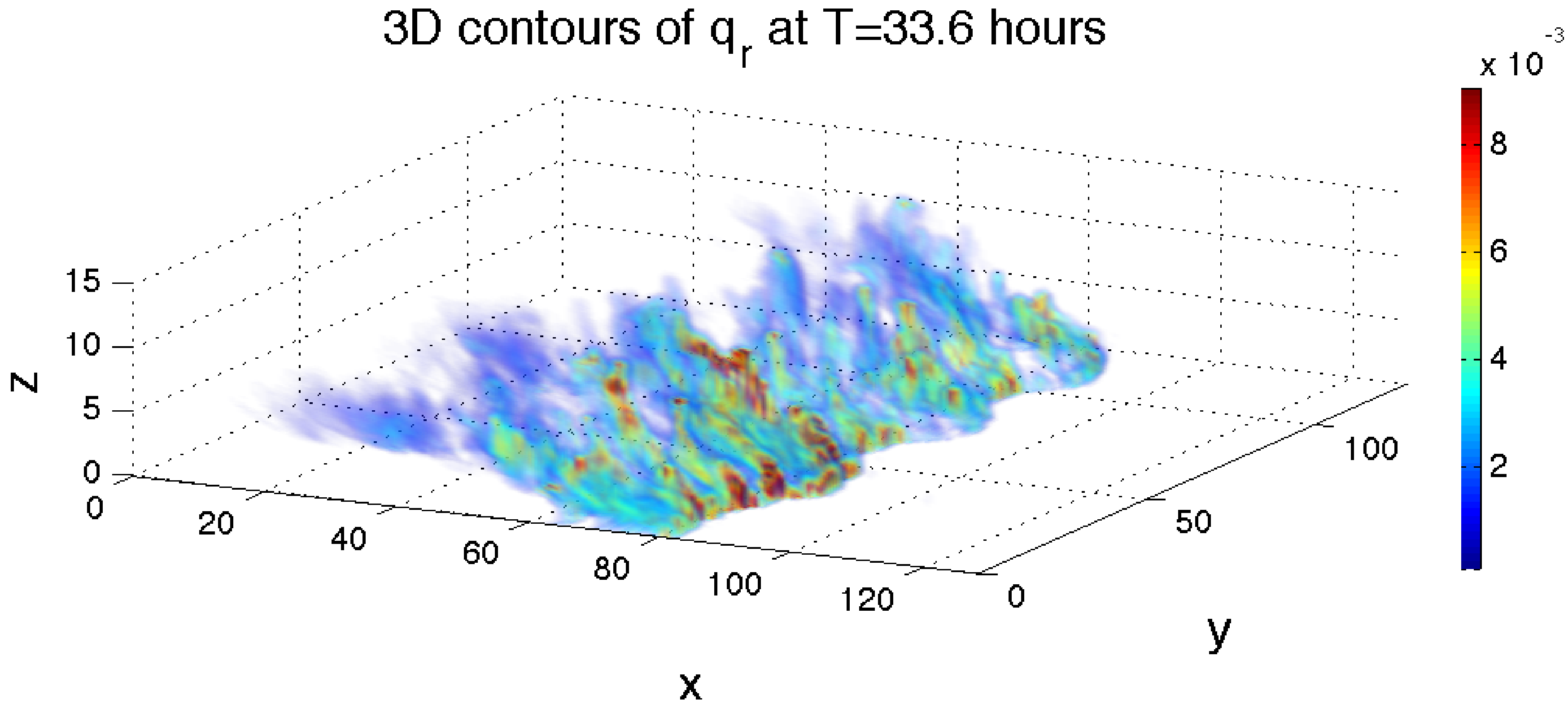
$$\frac{Dw}{Dt} = -\frac{\partial p}{\partial z} + g\left(\frac{\theta}{\theta_o} + \varepsilon_o q_v - q_r\right)$$

Contours of Water Anomaly ($128 \times 128 \times 15$ km)



Excess water, e.g. at low altitudes, leads to a precipitating updraft

A FARE Tropical Squall Line: 3D Contours of Rain Water



Hernandez-Duenas, Majda, S, Stechmann (2014)

Related Literature

Emanuel, Fantini, Thorpe: 1987; 2-layer, semi-geostrophic; ascending (descending) saturated (dry) air; baroclinic instability

Lapeyre, Held: 2004; 2-layer QG, turbulence study; parameterization of precip/latent heat release; jets (cyclonic vortices) dominate for weak (strong) latent heat release

Booth, Polvani, O'Gorman, Wang: 2015; extend dry theories using effective static stability; moisture $==>$ reduced stability parameterized by area fraction of upward motion

FARE QG for Scales larger than 1,000 km:

$$\frac{D\mathbf{u}}{Dt} + f\hat{\mathbf{z}} \times \mathbf{u} = -\nabla\left(\frac{p}{\rho_0}\right) + \hat{\mathbf{z}}\,(b_u H_u + b_s H_s), \qquad \nabla \cdot \mathbf{u} = 0$$

$$\frac{Dq_t}{Dt} + V_T \frac{\partial q_r}{\partial z} = 0, \qquad \frac{D\theta_e}{Dt} = 0, \qquad \theta_e = \theta + (L_v q_v / c_p)$$

$$b_u = g\left[\frac{\theta_e}{\theta_o} + \left(R_{vd} - \frac{L_v}{c_p \theta_o}\right) q_t\right]$$

$$b_s = g\left[\frac{\theta_e}{\theta_o} + \left(R_{vd} - \frac{L_v}{c_p \theta_o} + 1\right) q_{vs}(z) - q_t\right]$$

$H_u = 1$, $\quad H_u = 0$ unsaturated, $\quad H_s = 1 - H_u$

Distinguished Limit; Non-dimensional Parameters

$$Ro = \epsilon, \quad Fr = \frac{L}{L_d} Ro = O(\epsilon)$$

$L_d = NH/f$ is different for dry, unsaturated and saturated!

stable: q_t rapidly decreasing, θ_e rapidly increasing with z

$$G_M = -\frac{L_v}{c_p} \frac{d\tilde{q}_t/dz}{d\tilde{\theta}_e/dz} = O(1), \quad V_r = O(1)$$

All variables are expanded:

$$f = f^{(0)} + \epsilon f^{(1)} + \epsilon^2 f^{(2)} + \cdots$$

Lowest-Order Geostrophic, Hydrostatic Balance

$$u^{(0)} = -\frac{\partial\psi}{\partial y}, \quad v^{(0)} = \frac{\partial\psi}{\partial x}, \quad \zeta^{(0)} = \nabla_h^2\psi$$

$$b_u^{(0)} H_u + b_s^{(0)} H_s = \frac{L}{L_d}\frac{\partial\psi}{\partial z}$$

where ψ is the pressure.

Next-Order Correction: PV_u formulation of PQG

$$PV_u \equiv \nabla_h^2 \psi + \frac{L}{L_{du}} \frac{\partial b_u^{(0)}}{\partial z}$$

$b_u^{(0)}$ defined in both unsaturated and saturated regions

$$\frac{D_h^{(0)} PV_u}{Dt} = -\frac{L}{L_{du}} \hat{\mathbf{z}} \times \nabla_h \frac{\partial \psi}{\partial z} \cdot \nabla_h b_u^{(0)} + \frac{\partial}{\partial z} \left(V_r \frac{L}{L_{du}} \frac{\partial q_r^{(0)}}{\partial z} \right)$$

coupled to a thermodynamic equation involving q_t

Need jump conditions at phase boundaries

Iterative PV_u inversion to find ψ, H_u, H_s

A nonlinear elliptic equation:

$$\nabla_h^2\psi + \frac{L^2}{L_{du}^2}\frac{\partial}{\partial z}\left[H_u\frac{\partial\psi}{\partial z}\right] +$$

$$+D_M\frac{L^2}{L_{du}^2}\frac{\partial}{\partial z}\left[H_s\left(\frac{\partial\psi}{\partial z} + \frac{L_{du}}{L}(M - D_M q_{vs})\right)\right] = PV_u.$$

Old known locations of phase boundaries serve as the initial guess of iteration; $D_M = 1 + G_M > 1$ constant; $M = q_t + G_M\theta_e$.

Eady Baroclinic Instability (either phase; away from phase boundaries)

$$\psi = -\overline{U}zy + \psi'(x, y, t)$$

(zonal flow with vertical shear)

$$b^{(0)} = -(L/L_d)\overline{U}y + b'(x, y, z, t)$$

(temperature decreasing from equator to pole)

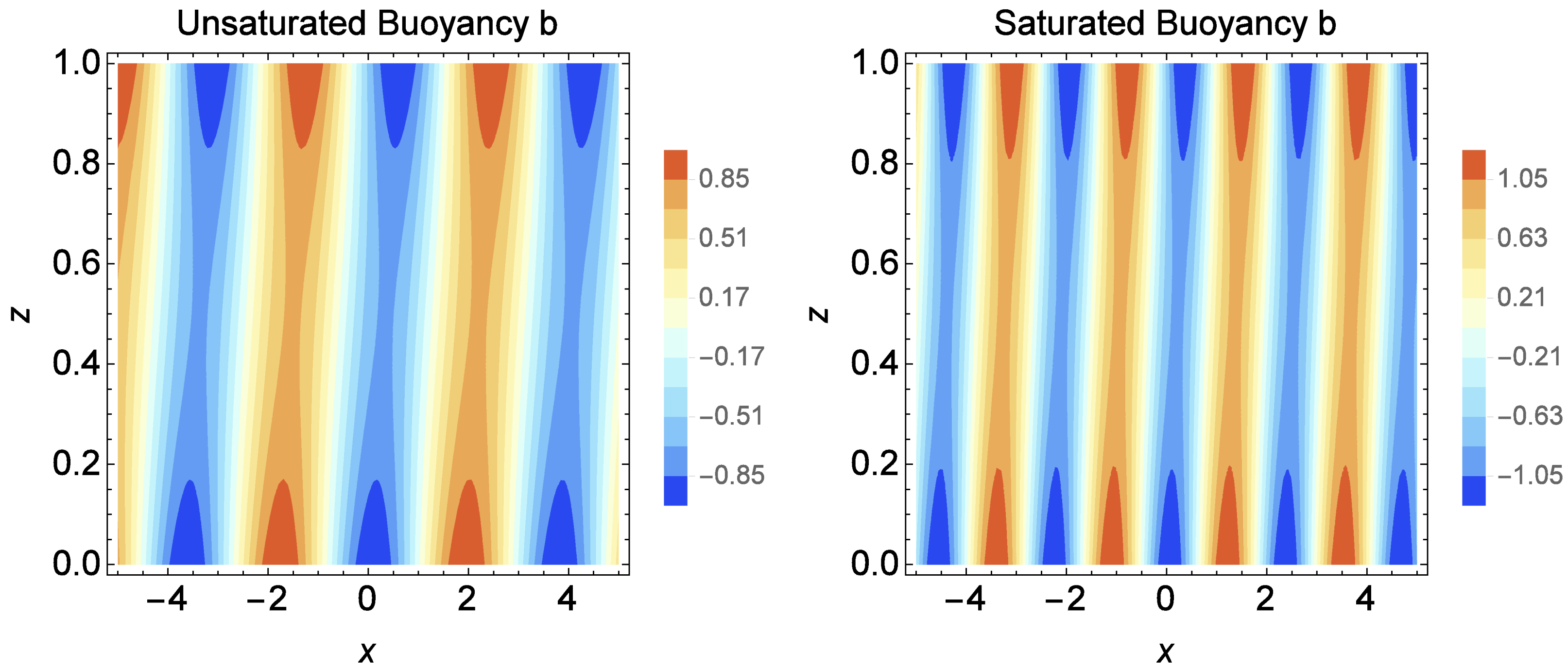
New feature (optional)

$$q_t^{(0)} = -\overline{Q}y + q_t'(x, y, x, t)$$

(water decreasing from equator to pole)

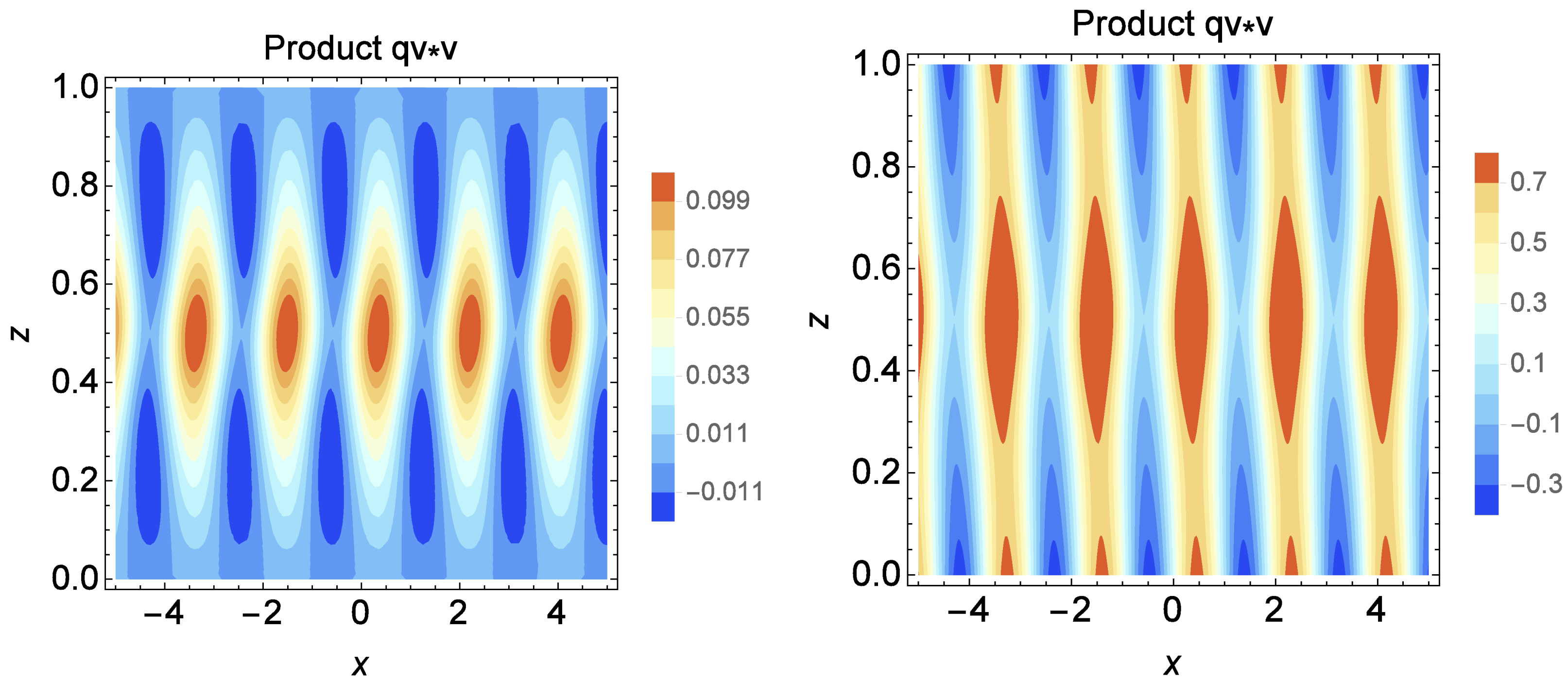
$$\psi' = \mathrm{Re}[\Psi(z)\exp(i(kx+ly-\omega t))], \quad q_t' = \mathrm{Re}[Q(z)\exp(i(kx+ly-\omega t))]$$

Comparison of Eady Anomalies; Buoyancy



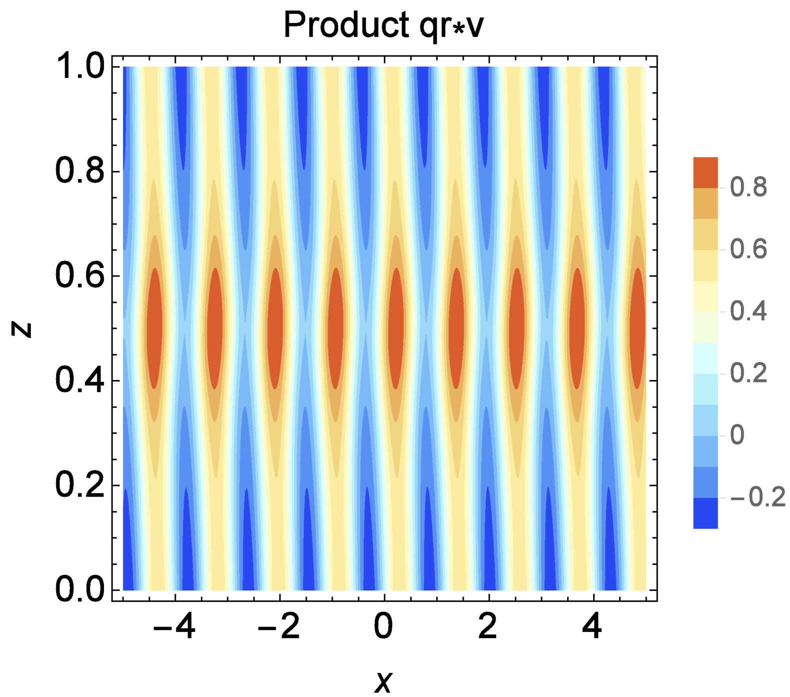
Most unstable mode of non-dimensional buoyancy anomaly (normalized by energy); Left (Right): Unsaturated (Saturated) buoyancy anomaly. Saturated wavelength 25% smaller.

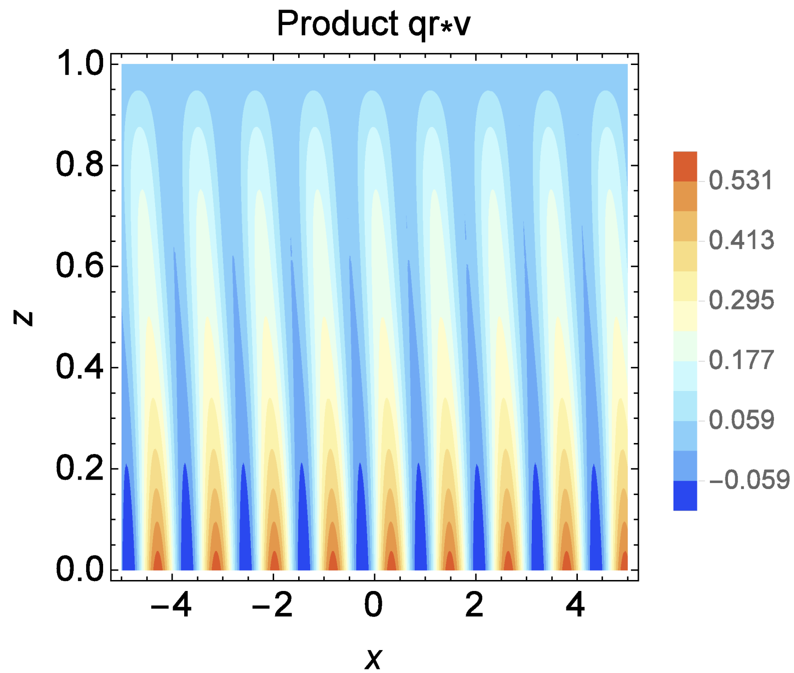
New! Fluxes of Water Vapor (unsaturated)



Meridional flux of water vapor anomaly; Left (Right): Eady background water vapor zero (decreasing) with latitude.

New! Fluxes of Liquid Water (saturated)





Meridional flux of rain water anomaly; Background rain decreasing with latitude; Left: $V_T = 0$; Right: $V_T = O(1)$.

Current/Future Work

- Simple exact solutions with a phase change,

e.g. saturated cold core transitioning to unsaturated outer core; axisymmetric

- Numerical methods for accurate/efficient computation of large-scale flow with evolving phase boundaries

(involves inversion of a nonlinear elliptic problem with unknown phase boundaries)